DOCUMENT RESUME

ED 088 708 SE 017 488

AUTHOR	Higgins, Jon L., Ed.
TITLE	Investigations in Mathematics Education, Volume 6 Number 4.
INSTITUTION	Ohio State Univ., Columbus. Center for Science and Mathematics Education.
PUB DATE	73
NOTE	59p.
AVAILABLE FROM	Ohio State University, Center for Science and Mathematics Education, 244 Arps Hall, Columbus, Ohio 43210 (Subscription, $6.00 year, $1.75 single copy)
EDRS PRICE	MF-$0.75 HC-$3.15
DESCRIPTORS	*Cognitive Development; *Concept Formation; Evaluation; *Instruction; Laboratories; *Mathematics Education; *Research Reviews (Publications); Teacher Education; Teaching Techniques; Textbook Research

ABSTRACT

Expanded abstracts and critical analyses are given for each of 14 research articles. Studies on instructional strategies or methods are reported in five reviews; three report studies concerned with concept attainment; three review studies in measurement of cognitive abilities; one study is on the effect of a laboratory for college freshman math students; one is on the change of prospective elementary teachers given geometric enrichment exercises; and one is on evaluation techniques for textbocks. A list is given of all mathematics education research reports included in ERIC references (RIE and CIJE) the previous quarter. (JP)

ED 088708

Vol. 6, No. 4 Autumn 1973

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INVESTIGATIONS IN MATHEMATICS EDUCATION

INVESTIGATIONS IN MATHEMATICS EDUCATION

Expanded Abstracts and Critical Analyses of Recent Research

Center for Science and Mathematics Education
The Ohio State University
in cooperation with
the ERIC Science, Mathematics and
Environmental Education Clearinghouse

SE 017 488

INVESTIGATIONS IN MATHEMATICS EDUCATION

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Published quarterly by

The Center for Science and Mathematics Education
The Ohio State University
1945 North High Street
Columbus, Ohio 43210

With the cooperation of the ERIC Science, Mathematics, and Environmental Education Information Analysis Center

Subscription Price: $6.00 per year. Single Copy Price: $1.75
Add 25¢ for Canadian mailings and 50¢ for foreign mailings.

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from the Editor

This issue continues the listing of mathematics education research reports listed in Research in Education and Current Index to Journals in Education during the past quarter. Since beginning this listing in the Spring, 1973 issue we have had several requests to include author's names in the listing. We have included those names, beginning with this issue.

Readers should also be aware that much additional information about documents in the Research in Education list can be easily obtained by using the ED access number and the appropriate issue of RIE. An abstract of each document, together with research grant information and a list of appropriate ERIC descriptors can be found in the "Document Resume" section.

Although Current Index to Journals in Education does not carry abstracts of articles, many of the articles listed are accompanied by short sentence annotations. Where annotations are not included, the analyst has judged that the list of ERIC descriptors fairly describes the content of the articles.

Although we believe that this listing provides an excellent way to keep abreast of current research in mathematics education, we recognize that the list has some shortcomings. For an analysis of these shortcomings, the reader is referred to the Spring, 1973 issue of Investigations in Mathematics Education.

Jon L. Higgins
Editor

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN RESEARCH IN EDUCATION
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ED 073 331 Kent, William P., Test Data on Adult Basic Education Students. 27p. MF and HC available from EDRS.

ED 073 583 Funk, Kerri L. and Tseng, M.S., Effects of Classification Exposure upon Numerical Achievement of Educable Mentally Retarded Children. 12p. MF and HC available from EDRS.

ED 073 592 Fountain Valley School District, California, Handicapped Children in the Regular Classroom. 26p. MF and HC available from EDRS.

ED 073 676 Rudner, Lawrence M., The Effect of Student-Produced Filmstrips on Mathematics Achievement in Grade Eight. 44p. MF and HC available from EDRS.

ED 073 678 Fisher, Maurice D., Strategies for Developing Reciprocity Between Educational Researchers and School Personnel: Providing Teachers with Feedback on Students' Computer Assisted Instruction Performance. 34p. MF and HC available from EDRS.

ED 073 839 Weiner, Susan L., On the Development of "More" or "Less." 32p. MF and HC available from EDRS.

ED 073 917 Watson, Carlos and others, A Study of the Required Mathematics Content Courses for Undergraduate Elementary Teachers in the United States. 8p. MF and HC available from EDRS.

ED 073 926 Beardslee, Edward C. and Jerman, Max E., Linguistic Variables in Verbal Arithmetic Problems. 26p. MF and HC available from EDRS.

ED 073 941 Bourne, Lyle E., Jr., Long-Term Retention of Simple Concepts. 19p. MF and HC available from EDRS.

ED 073 942 Feldman, Katherine V., The Effects of Number of Positive and Negative Instances, Concept Definition, and Emphasis of Relevent Attributes in the Attainment of Mathematical Concepts. 120p. MF and HC available from EDRS.

ED 073 943 McLeod, Douglas B., The Effectiveness of an Inservice Program for Implementing an Activity Approach to Learning Mathematics in the Elementary School. 194p. MF and HC available from EDRS.

ED 073 944 Sawyer, Ray C., Evaluation of Alternative Methods of Teaching Subtraction of Integers in Junior High School. Final Report. 170p. MF and HC available from EDRS.

ED 074 054 Campbell, Lloyd P. and Williamson, John A., Problems in Student Teaching: Academic versus Non-Academic Subjects. 6p. MF and HC available from EDRS.

ED 074 110 Ellis, E.N., Survey of Achievement in Mathematics in Year Six of Vancouver Schools, May 29 - June 2, 1972. 15p. MF and HC available from EDRS.

ED 074 113 Ellis, E.N., Survey of Achievement in Arithmetic in Year Three of Vancouver Schools, May 29 - June 2, 1972. Research Report. 9p. MF and HC available from EDRS.

ED 074 154 Knipe, Walter H. and Krahmer, Edward F., An Application of Criterion Referenced Testing. 19p. MF and HC available from EDRS.

ED 074 189 Tallmadge, G. Kasten, An Analysis of the Relationship Between Reading and Mathematics Achievement Gains and Per-pupil Expenditures in California Title I Projects, Fiscal Year 1972. Final Report. 46p. MF and HC available from EDRS.

ED 074 305 Olson, LaVene A., An Evaluation of Elementary Career Education Based on Language Achievement, Mathematics Achievement, and Occupational Awareness in Lincoln County, West Virginia. Volume V of Volume I. 86p. MF and HC available from EDRS.

ED 074 441 Hawaii State Dept. of Education, Honolulu. Office of Instructional Services, Item Study Summary Report Sequential Tests of Educational Progress (STEP); Reading, Mathematics and Writing for Grades 4, 6, 8, 10 and 12 by State and District School Year 1970-1971. Research Report No. 77. 268 p. MF and HC available from EDRS.

ED 074 473 Bremmer, Barbara L., Students Helping Students Program, 1971-72. Final Report. 46p. MF and HC available from EDRS.

ED 074 706 Research in Learner-Controlled Computer-Assisted Instruction. Annual Report. (Final) 1 May 70 - 30 April 71. 31p. Document not available from EDRS. Available from - National Technical Information Service, Springfield, Virginia 22151.

ED 074 734 Durall, Edwin P., A Feasibility Study: Remediation By Computer Within a Computer-Managed Instruction Course in Junior High School Mathematics. 79p. MF and HC available from EDRS.

ED 074 747 Ash, Michael J. and Sattler, Howard E., A Video Tape Technique for Assessing Behavioral Correlates of Academic Performance. 18p. MF and HC available from EDRS.

ED 075 170 Sobelman, Marilyn, The Relationship of Secondary School Teachers' Subject Specialization, Teaching Level, and Reaction to Selected Teaching Situations. 188p. Not available from EDRS. Available from University Microfilms (71-24,832).

ED 075 174 Kolebas, Patricia, The Effects on the Intelligence, Reading, Mathematics, and Interest in Science Levels of Third Grade Students Who Have Participated in Science - A Process Approach Since First Entering School. 113p. Not available from EDRS. Available from University Microfilms (72-7123).

ED 075 175 Wheeler, Otis V., Jr., The Relative Effectiveness of Two Different Mathematics and Science Programs Used With Disadvantaged and Non-Disadvantaged First Grade Children. 119p. Not available from EDRS. Available from University Microfilms (72-10,566).

ED 075 180 Schater, Larry E., Inducing Stage III Seriation Capabilities in Kindergarten Children Through Cue Fading and Reinforcement. 253p. Not available from EDRS. Available from University Microfilms (72-22,282).

ED 075 183 Easton, Stanley E., The Relation Between Certain High School Course Patterns and Achievement in First Freshman Courses in English, Social Science, Mathematics, and Natural Science at Louisiana State University. 92p. Not available from EDRS. Available from University Microfilms (71-6562).

ED 075 184 Kamps, Kenneth G., An Investigation of Portions of a Model for Acquisition of Conservation and Measurement of Length Based on Performance of Selected Second Grade Children on Six Piaget-Type Tasks. 99p. Not available from EDRS. Available from University Microfilms (71-5769).

ED 075 185 Smith, Dan F., A Study of the Relationship of Teacher Sex to Fifth Grade Boys' Sex Role Preference, General Self Concept, and Scholastic Achievement in Science and Mathematics. 117p. Not available from EDRS. Available from University Microfilms (71-4312).

ED 075 186 Cleminson, Ronald W., A Comparative Study of Three Fifth Grade Classrooms on Five Selected Piaget Type Tasks Dealing with Science Related Concepts. 140p. Not available from EDRS. Available from University Microfilms (71-5725).

ED 075 193 Sowder, Larry K. and others, Advance Organizers and Objectives in Teaching Mathematics. 17p. MF and HC available from EDRS.

ED 075 194 Shumway, Richard J., Positive Versus Positive and Negative Instances and the Acquisition of the Conjunctive Concepts of Distributivity and Homomorphism. 21p. MF and HC available from EDRS.

ED 075 196 Smith, Gerald E., Relationships Between Cognitive Style and Instructional Treatment Among Prospective Elementary School Teachers. 5p. MF and HC available from EDRS.

ED 075 198 Martin, J. L., An Investigation of the Development of Selected Topological Properties in the Representational Space of Young Children. 35p. MF and HC available from EDRS.

ED 075 199 Weaver, J.F. and others, Some Factors Associated with Pupils' Performance on Examples Involving Selected Variations of the Distributive Idea. 50p. MF and HC available from EDRS.

ED 075 260 Bassler, Otto C. and others, Comparison of Two Instructional Strategies for Teaching the Solution to Verbal Problems. Final Report. 69p. MF and HC available from EDRS.

ED 075 264 Suppes, Patrick, Mathematical Models of Elementary Mathematics Learning and Performance. Final Report. 30p. MF and HC available from EDRS.

ED 075 369 Greabell, Leon C., Jr., A Quasi-Experimental Study Designed to Evaluate the Effectiveness of a Student-Goal-Determined Course in Mathematics Education. 10p. MF and HC available from EDRS.

ED 075 512 Kulm, Gerald, A Mathematics Self-Concept Test. 11p. MF and HC available from EDRS.

ED 075 969 Fox, Lynn H. and Stanley, Julian C., Educational Facilitation for Mathematically and Scientifically Precocious Youth. 24p. MF and HC available from EDRS.

ED 075 971 Keating, Daniel P. and Stanley, Julian C., Discovering Quantitative Precocity. 11p. MF and HC available from EDRS.

ED 076 039 Hall, Keith A. and others, Inservice Mathematics Education for Elementary School Teachers Via Computer-Assisted Instruction (Dryden). Interim Report. 61p. MF and HC available from EDRS.

ED 076 042 Hall, Keith A. and others, Inservice Mathematics Education for Elementary School Teachers Via Computer-Assisted Instruction (Gladeville). Interim Report. 72p. MF and HC available from EDRS.

ED 076 043 Hall, Keith A. and others, Inservice Mathematics Education for Elementary School Teachers Via Computer-Assisted Instruction (California). Interim Report. 59p. MF and HC available from EDRS.

ED 076 053 Hall, Keith A. and others, The Development, Implementation and Evaluation of a Pilot Program of Computer-Assisted Instruction for Urban High Schools: General Mathematics and Algebra I. Summary Report. 25p. MF and HC available from EDRS.

ED 076 306 Smith, Charlotte E.O., The Structure of Intellect Protocol Analysis System: A Technique for the Investigation and Quantification of Problem Solving Processes. 132p. Not available from EDRS. Available from University Microfilms (72-7745).

ED 076 316 Stone, Gwen E.G., Three Approaches to Assessing the Conservation of Weight Concept. 129p. Not available from EDRS. Available from University Microfilms (72-20,003).

ED 076 319 Kraemer, Ruth A., The Effects of the Cluster Plan on Mathematics and Science Students' Achievement Scores in the Oklahoma City Public Schools, 1970-1971. 93p. Not available from EDRS. Available from University Microfilms (72-22,137).

ED 076 323 Graves, Avis J.R., The Attainment of Conservation of Mass, Weight, and Volume in Minimally Educated Adults. 80p. Not available from EDRS. Available from University Microfilms (72-21,067).

ED 076 340 Little, Richard A., A Taxonomic Approach to Measuring Achievement in Mathematics 223 - Geometry for Elementary Teachers. 206p. Not available from EDRS. Available from University Microfilms (72-15,945).

ED 076 341 Hopkins, Layne V., Toward a Theory of Sequencing: Study 3-2: An Exploration of Transitivities Formulated From a Set of Piagetian-Derived Operations and Their Implications in Traversing Learning Hierarchies. 111p. Not available from EDRS. Available from University Microfilms (72-13,873).

ED 076 342 Sawada, Daiyo, Toward a Theory of Sequencing: Study 3-1: Curriculum Hierarchies and the Structure of Intelligence: A Strategy of Organizing Instructional Objectives into Mathematical Systems Employing Basic Piagetian Constructs. 150p. Not available from EDRS. Available from University Microfilms (72-13,926).

ED 076 343 Sumagaysay, Lourdes S., The Effects of Varying Practice Exercises and Relating Methods of Solution in Mathematics Problem Solving. 148p. Not available from EDRS. Available from National Library of Canada, 395 Wellington Street, Ottawa, Ontario K1A ON4, Canada.

ED 076 369 Loh, Elwood L., The Effect of Behavioral Objectives on Measures of Learning and Forgetting on High School Algebra. 292p. Not available from EDRS. Available from University Microfilms (72-20,259).

ED 076 370 Silbaugh, Charlotte V., A Study of the Effectiveness of a Multiple-Activities Laboratory in the Teaching of Seventh Grade Mathematics to Inner-City Students. 69p. Not available from EDRS. Available from University Microfilms (72-19,733).

ED 076 371 Johanson, Emma J.D., A Ninth Grade Piagetian Mathematics Curriculum. 206p. Not available from EDRS. Available from University Microfilms (72-20,180).

ED 076 372 May, Daryle C., An Investigation of the Relationship Between Selected Personality Characteristics of Eighth-Grade Students and Their Achievement in Mathematics. 89p. Not available from EDRS. Available from University Microfilms (72-21,080).

ED 076 373 Troutman, Andria P., Development of a Specific Cognitive Observation System for the Analysis of Mathematics Teaching. 142p. Not available from EDRS. Available from University Microfilms (72-21,110).

ED 076 374 Stephenson, Carl G., A Comparison of the Postulational Structur of the Synthetic, Transformation and Vector Approaches to Plane Geometry. 120p. Not available from EDRS. Available from University Microfilms (72-23,114).

ED 076 375 Koch, Dale R., Concept of Self and Mathematics Achievement. 136p. Not available from EDRS. Available from University Microfilms (72-23,622).

ED 076 389 Mowbray, John S., Jr. and Seager, G. Bradley, Jr., Sequencing of Instructional Activities. 11p. MF and HC available from EDRS.

ED 076 391 Milner, Stuart, The Effects of Computer Programming on Performance in Mathematics. 41p. MF and HC available from EDRS.

ED 076 392 Fennema, Elizabeth, Mathematics Learning and the Sexes: A Review. 24p. MF and HC available from EDRS.

ED 076 393 Webb, Leland F. and others, Attainment of Mathematical Concepts by Preservice Elementary School Teachers. 13p. Not available from EDRS. Available from ERIC/SMEAC, 400 Lincoln Tower, 1800 Cannon Drive, Columbus, Ohio 43210 (on loan).

ED 076 394 Webb, Leland F. and Sherrill, James M., The Effects of Differing Presentations of Mathematical Word Problems Upon the Achievement of Preservice Elementary Teachers. 12p. Not available from EDRS. Available from ERIC/SMEAC, 400 Lincoln Tower, 1800 Cannon Drive, Columbus, Ohio 43210 (on loan).

ED 076 414 Bright, George W., Geometric Problem Solving Abilities of Children in the Primary Grades. 27p. MF and HC available from EDRS.

ED 076 423 Peterson, John C. and Hancock, Robert R., Developing Mathematical Materials for Student's Cognitive Style. 34p. MF and HC available from EDRS.

ED 076 424 Beattie, Ian D. and others, The Relationship of Achievement and Attitudes Towards Mathematics in the Elementary School: A Longitudinal Study. 15p. MF and HC available from EDRS.

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ED 076 431 Segalla, Angelo, Using Structural Variables to Predict Word - Problem Solving Difficulty for Junior College Arithmetic Students. 115p. MF and HC available from EDRS.

ED 076 433 Cook, Blair, An Analysis of Arithmetic, Linguistic, and Algebraic Structural Variables That Contribute to Problem Solving Difficulty in Algebra Word Problems. 17p. MF and HC available from EDRS.

ED 076 434 King, Irv, Ikonic and Symbolic Representation. A Study of Mathematical Reasoning. 14p. MF and HC available from EDRS.

ED 076 437 Suydam, Marilyn N. and Weaver, J. Fred, Research on Mathematics Education (K-12) Reported in 1972. 80p. Not available from EDRS. Available from Center for Science and Mathematics Education, The Ohio State University, 244 Arps Hall, Columbus, Ohio 43210.

ED 076 632 Hughes, Hughie, Can ACT Scores be Used to Predict Prospective Elementary Teachers' Ability to Understand Concepts Used in New Mathematics Curricula? 9p. MF and HC available from EDRS.

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN JOURNALS AS INDEXED BY CURRENT INDEX TO JOURNALS IN EDUCATION
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EJ 073 794 Knight, Carlton W., II. "Doctoral Dissertation Research in Science and Mathematics Reported for Volume 31 of Dissertation Abstracts: Part II - Mathematics." School Science and Mathematics, v73 n2, pp121-46, Feb 73.

EJ 073 799 Nickson, M.T.; Smith, R.P. "An Investigation into Alternative Methods of Teaching Elementary Mathematics to Students of Economics." International Journal of Mathematical Education in Science and Technology, v4 n1, pp43-49, Jan-Mar 73.

EJ 074 849 Klein, Paul A. "Study 1-1: An Exploration of Selected Relationships Among the Enactive, Iconic, and Symbolic Modes of Representation." Journal for Research in Mathematics Education, v4 n2, pp94-103, Mar 73.

EJ 075 101 Maffei, Anthony C. "Reading Analysis in Mathematics." Journal of Reading, v16 n7, pp546-49, Apr 73.

EJ 075 270 Lewis, D.G.; Ko, Peng-Sim. "Personality and Performance in Elementary Mathematics with Special Reference to Item Type." British Journal of Educational Psychology, v43 pt1, pp24-34, Feb 73.

EJ 075 346 Romberg, Thomas A.: Wilson, James W. "The Effect of an Advance Organizer, Cognitive Set, and Post Organizer on the Learning and Retention of Written materials." Journal for Research in Mathematics Education, v4 n2, pp68-76, Mar 73.

EJ 075 347 Peterson, John C.; and others. "The Effect of Organizers and Knowledge of Behavioral Objectives on Learning a Mathematical Concept." Journal for Research in Mathematics Education, v4 n2, pp76-84, Mar 73.

EJ 075 348 Heimer, Ralph T.; Lottes, John J. "Toward a Theory of Sequencing: An Integrated Program of Research. The Theoretical Model and a Synopsis of the First Two Years of the Research Program." Journal for Research in Mathematics Education, v4 n2, pp85-93, Mar 73.

EJ 075 349 Farris, Dan C. "Study 1-2: An Exploration of Selected Relationships Among the Enactive, Iconic, and Symbolic Modes of Representation." Journal for Research in Mathematics Education, v4 n2, pp104-105, Mar 73.

EJ 075 350 Paquette, Gerald A. "Study 1-3: An In-Depth Exploration of the Role of Iconic Representations in the Study of Congruence of Triangles." Journal for Research in Education, v4 n2, pp105-110, Mar 73.

EJ 075 351 Bowers, Robert Garth. "Study 1-4: The Effect of Triangle-Pair Configuration Variation on Achievement of Selected Classes of Instructional Objectives in Plane Geometry." Journal for Research in Mathematics Education, v4 n2, pp110-113, Mar 73.

EJ 075 352 Hirschbuhl, John J. "Study 1-5: An Exploration of Selected Transitivity and Conjunctive Relationships Among the Enactive, Iconic, and Symbolic Modes of Representation." Journal for Research in Mathematics Education, v4 n2, pp113-115, Mar 73.

EJ 075 353 Hostetler, Robert P. "Study 2-1: An Exploration of the Effect of Selected Sequence Variables on Student Choice in the Use of Algorithms." Journal for Research in Mathematics Education, v4 n2, pp115-118, Mar 73.

EJ 075 354 Sawada, Daiyo. "Study 3-1: An Assessment of a Selected Set of Piagetian-Derived Operators for the Generation of Effective Learning Hierarchies." Journal for Research in Mathematics Education, v4 n2, pp118-121, Mar 73.

EJ 075 355 Hopkins, Layne V. "Study 3-2: An Exploration of Transitivities Formulated From a Set of Piagetian-Derived Operations and Their Implications in Traversing Learning Hierarchies." Journal for Research in Mathematics Education, v4 n2, pp121-123, Mar 73.

EJ 075 357 O'Brien, Thomas C. "Logical Thinking in Adolescents." Educational Studies in Mathematics, v4 n4, pp401-428, Dec 72.

EJ 077 048 Sherrill, James M. "The Effects of Different Presentations of Mathematical Word Problems Upon the Achievement of Tenth Grade Students." School Science and Mathematics, v73 n4, pp277-282, Apr 73.

EJ 077 050 Smith, Robert F. "Diagnosis of Pupil Performance on Place-Value Tasks." Arithmetic Teacher, v20 n5, pp403-408, May 73.

EJ 077 051 Waits, Bert K.; Elbrink, Larry C. "Student Evaluation of Mathematics Instruction." Two-Year College Mathematics Journal, v4 n2, pp59-66, Spr 73.

EJ 077 052 Perry, Donald. "A Study, Using CUPM Recommendations as Criteria, of the Academic Preparation of Two-Year College Teachers." Two-Year College Mathematics Journal, v4 n2, pp67-71, Spr 73.

EJ 077 053 Behr, Anthony N. "Achievement, Aptitude and Attitude in Mathematics." Two-Year College Mathematics Journal, v4 n2, pp72-74, Spr 73.

EJ 076 732 Gay, Lorraine R. "Temporal Position of Reviews and its Effect on the Retention of Mathematical Rules." Journal of Educational Psychology, v64 n2, pp171-82, Apr 73.

ED 075 260 SE 016 095
COMPARISON OF TWO INSTRUCTIONAL STRATEGIES FOR TEACHING THE SOLUTION TO VERBAL PROBLEMS. FINAL REPORT. Bassler, Otto C.; Beers, Morris I.; Richardson, Lloyd I.; George Peabody College for Teachers, Nashville, Tennessee. Spons Agency--Office of Education (DHEW), Washington, D.C. National Center for Educational Research and Development. Pub Date Dec. 72, Note--69p, EDRS Price MF-$0.65 HC-$3.29.
Descriptors--*Algebra, *Instruction, Mathematics Education, Multimedia Instruction, *Problem Solving, *Research, *Secondary School Mathematics, Symbols (Mathematics), Teaching Machines

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James K. Bidwell, Central Michigan University.

1. Purpose

Two strategies for teaching the solution of verbal problems were compared. In one strategy, the Polya Method (PM), students read for understanding, then planned, carried out, and checked the solution. In the other strategy, the Dahmus Method (DM), students did a direct, pure, piece-meal, and complete translation of the problem into mathematical statements, then solved and checked the solution.

2. Rationale

Students do not have a good method for attacking verbal problems. Teachers do not seem to have effective strategies for teaching verbal problems to students. Teachers avoid general strategies in favor of specific problem types.

Polya advocates a general strategy which is heuristic in nature; students must analyze the problem and look for analogies to find a solution. Another general strategy is advocated by Dahmus. Students translate the problem into several equations, use several variables, in a direct, pure, piece-meal, and complete manner. These strategies are diametrically opposed.

Much research has been done comparing different strategies for solving problems. Different strategies studied include the step method, the graphical method, the wanted-given method, and the action sequence method. No studies have compared the step method with a translation method.

3. Research Design and Procedure

Instructional materials for each strategy were prepared as a sight-sound presentation. The presentations were controlled by the investigators. A pilot study was done using seventh and eighth grade students. Considerable revision was done and the final study used ninth

grade students. There were seven 40 minute periods of instruction. Selection of variables to represent unknowns and solution of linear equations was taught the first two days to all students. The remainder of instruction was spent on the particular strategy.

The Orleans-Hanna Prognosis Test was used as a measure of ability in Algebra I. A posttest of ten problems was given all subjects the day following the seventh instructional period. A similar retention test was given unannounced four weeks later. Students were instructed to find an equation and to solve the problem. The tests were scored for both items by two investigators on a right-wrong basis, yielding a maximum score of 20.

4. Findings

Table 1 shows the mean scores on the posttest and retention tests. ANOVA showed that the PM group scored significantly higher than the DM group ($p < .05$) on the equation criterion ($\bar{x}$). Retention scores were significantly higher ($p < .05$) than posttest scores on the solution criterion ($\bar{y}$). There were also significant differences between ability groups.

5. Interpretation

Within this design the PM is preferable to the DM. Limiting conditions require that conclusions should be accepted with caution. Performances were not outstanding in either method. Means ranged from 2% to 65% of possible score on the equation criterion and from 25% to 75% on the solution criterion. Low attainment may have suppressed differences between groups. The carefully sequenced instruction design does not permit open questioning and teacher-pupil interaction.

Table 1
Posttest and Retention Test Means of the Equation Criterion Scores ($\bar{x}$) and the Problem Solution Scores ($\bar{y}$).

	Treatment			
	Polya		Dahmus	
Ability Level	$\bar{x}$	$\bar{y}$	$\bar{x}$	$\bar{y}$
Posttest				
High	12.5	12.5	9.6	12.5
Intermediate	8.3	9.5	2.1	7.6
Low	4.9	6.8	1.4	5.0
Retention Test				
High	13.1	14.4	8.6	12.0
Intermediate	9.6	9.0	1.9	9.3
Low	3.9	8.6	0.4	7.1

(n per cell = 8) (maximum score = 20)

Students solve problems before grade nine, usually by a method that reinforces the PM. This may account for some observed differences. DM subjects did learn to translate problems, but failed to produce an acceptable single equation.

The resistance to forgetting (in fact, improved scores), may have been influenced by the problem-solving nature of Algebra I. No additional instruction was done on verbal problems. Analysis of the test problems showed that the gains were made on the more complex problems on the test.

It is recommended that students receive instruction similar to the Polya Method or possibly a synthesis of both methods.

Abstractor's Notes

Three main comments seem to be in order. First the small sample split six ways with a wide range of ability [lowest group mean 60.1 (s.d. 13.5), highest group mean 85.9 (s.d. 2.1)] makes extrapolation very unreliable. Significant differences are not surprising.

Second, time limits for the tests are not mentioned. Assuming a 40 minute class period, ten problems for which to write equations and solve are too many. This is shown by the larger number of solutions than equations. Writing equations (particularly by the DM) is time consuming. The test problems are not simple enough. The abstractor believes that a 40 minute time limit on the tests would invalidate the results of this research.

Third, the DM does not yield a single equation to solve a problem. A series of equations using several variables and requireing substitution is the usual paradigm for this method. A single equation criterion seems out of place. The low means of the DM group are not surprising.

James K. Bidwell
Central Michigan University

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EJ 071 812 270 SE 507 854

THE EFFECTS OF A LABORATORY ON ACHIEVEMENT IN COLLEGE FRESHMAN MATHEMATICS.

Douthitt, Cameron, Two-Year College Mathematics Journal, v4 n1, pp55-59, W 73.

Descriptors--*College Mathematics, *Instruction, *Laboratories, *Mathematics Education, *Research, Achievement, Analytic Geometry, Attitudes

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Roland F. Gray, University of British Columbia.

1. Purpose

The purpose was to determine the effects of a mathematics laboratory on achievement, attitude and withdrawal-failure rate among first year college mathematics students.

(Only the effects on achievement were reported in this article.)

2. Rationale

The author noted that at a particular designated university the withdrawal-failure rate among first year mathematics students is approximately 30% while for "risk" students, the rate is approximately 70%. Risk students were defined as those in the top 25% of their high school graduating classes who scored less than 450 on the mathematics section and total score of less than 800 on the Scholastic Aptitude Test. These students were from predominantly black schools. While their grades were high in secondary school, they tended not to succeed in college. It was assumed that unsatisfactory work in mathematics was one factor in their lack of success in college. The author conducted this study in an attempt to discover some means of helping these students improve their mathematics performance.

3. Research Design and Procedure

Control and experimental groups were established from students who registered for analytic geometry. Each group contained both risk and non-risk students. The experiemntal group was composed of 26 risk students who volunteered for a special summer orientation and 42 non-risk students. The risk students were not told of their risk status. These 68 students attended two sections of the course which met for lectures two days a week and one day for laboratories. Senior students served as tutors in the laboratories and met with groups of 5 or 6 students. The control group consisted of 112 risk and non-risk students; the remainder of those who registered for analytic geometry. These students attended regular lectures three times a week and had no laboratory experience. The experimental group was subdivided into two different groups according to type of laboratory; T-1 and T-2. The students in the T-1 group used only the text and in the laboratory worked problems with the aid of tutors. The T-2 students used a variety of materials including programmed texts, filmstrips, tapes, models and transparencies.

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Ten null hypotheses were tested by a point biserial correlation analysis to test for achievement differences between:

(a) experimental and control groups,
(b) categories within groups and across groups,
(c) the two laboratory procedures used (T-1 and T-2),
(d) categories within and across the T-1 and T-2 laboratory groups.

The criterion measures employed were scores on the Cooperative Mathematics Test--Analytic Geometry.

4. Findings

Statistically significant differences were found favoring:

(a) Non-risk students in the experiemntal group over non-risk students in the control group.
(b) Risk students in the experimental group over risk students in the control group.
(c) The experimental group over the control group.

No significant differences were found between:

(a) Risk and non-risk students in the experiemntal group.
(b) The two laboratory procedures, nor between risk and non-risk students within or across laboratories.

5. Interpretations

The author offered seven conclusions:

(a) A mathematics laboratory can produce higher achievement in freshman mathematics.
(b) Risk students can do acceptable work when given motivation and encouragement in a laboratory.
(c) Programmed texts seemed to be of greatest benefit.
(d) The type of laboratory did not seem to affect achievement.
(e) The laboratory does not seem to be necessary for all students. It seems best for those with a deficiency in mathematics.
(f) Expository teaching does not result in greater coverage as the laboratory groups covered more topics.
(g) Students and tutors who were in the experimental sections think a laboratory is beneficial.

Abstractor's Notes

It is difficult indeed to comment fairly and wisely on studies that are reported briefly and only in part. One assumes that much data are not reported which possibly could serve to clarify problems and doubts that often arise in trying to assess the value of such a study. With this in mind, however, some observations seem worthy of note as follows:

1. The risk students in the experimental group, it appears, were only those who volunteered for the special course and who were willing to attend extra meetings in the preceeding summer. Such a procedure would likely produce a biased sample with a built-in success striving factor predisposing this group to higher achievement.

2. The nonsignificant findings between risk and non-risk students in both groups would suggest that this particular group of risk students may have been atypical and thereby were a vitiating factor in this study. This possibility seems to have escaped the author's attention.

3. The evidence reported and the design features described leaves doubt that the laboratory was in fact causally related to achievement. Any superiority obtained could easily have resulted from such attendant variables as special attention, small group instruction, or some interpersonal relational factors. None of these variables seem to have been considered.

4. Several conclusions stated are not supported by data reported. One must suppose the existence of data other than gross opinion on which the conclusions could be justified.

5. This article like many others argues for a set of standardized procedures for reporting studies in mathematics education which will assure that relevant data are carefully and fully reported.

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ED 073 942 SE 015 849

THE EFFECTS OF NUMBER OF POSITIVE AND NEGATIVE INSTANCES, CONCEPT DEFINITION, AND EMPHASIS OF RELEVANT ATTRIBUTES IN THE ATTAINMENT OF MATHEMATICAL CONCEPTS.

Feldman, Katherine Vorwerk, Wisconsin Univ., Madison. Research and Development Center for Cognitive Learning. Pub Date Nov. 72, Note--120p.
EDRS Price MF-$0.65 HC-$6.58.
Descriptors--Cognitive Processes, *Concept Formation, Elementary School Mathematics, Geometric Concepts, *Learning, Learning Theories, *Mathematics Education, *Research

Abstract and Analysis Prepared Especially for I.M.E. by Richard J. Shumway, The Ohio State University.

1. Purpose

To study the relationship between the number of teaching instances, the presence or absence of a definition, and the presence or absence of an emphasis on the relevant attributes on the attainment of the concepts of bilateral symmetry, rotational symmetry, and translational symmetry.

2. Rationale

The majority of research on concept learning has been conducted in the laboratory under highly controlled conditions. Klansmeier's model of the cognitive operations involved in concept learning together with a taxonomy of variables thought to influence concept learning were used to generate this research.

3. Research Design and Procedure

All three experiments used a 3 x 4 factorial design with three levels of reading achievement and four types of lessons.

For Experiment I, the lesson types were concept lessons containing: (1) the rational set of both examples and non-examples; (2) the rational set of examples only; (3) two examples; and (4) placebo lessons. For Experiment II, definitions of the concepts were added to all the lessons. For Experiment III, the lesson types were identical to those for Experiment II except that relevant attributes were emphasized in lesson type (3). All treatments but the placebo dealt with bilateral symmetry, rotational symmetry, and translational symmetry.

The dependent measures were: (1) correct classification of examples and non-examples; (2) recognition of concept definitions; (3) knowledge of relationships among concepts; (4) overgeneralization classification errors; and (5) undergeneralization classification errors.

Subjects were sixth-graders, 79, 111, and 100 respectively. Treatment and test for each experiment took 35-50 minutes plus 10-20 minutes for a retention test two weeks later.

A 3 x 4 analysis of variance was performed on the data for each of the five dependent variables. When main effects were significant, pairwise comparisons were made using Tukey's procedure.

4. Findings

The main effect for the reading level stratification was significant over all experiments and all dependent variables. Good readers performed better than poor readers.

Other results indicate:

	Experiment		
Dependent Variable	I	II (defns. added)	III (type 3 att. emph.)
Correct Classification of Instances			
Immediate	1 > 4*	1,2,3 > 4	2,3 > 1,4
Retention		1,2,3 > 4	2,3 > 4
Recognition of Definitions			
Immediate		1,2,3 > 4	3,2 > 1,4
Retention		1,2,3 > 4	3 > 1,4
Knowledge of Relationships Among Concepts		3 > 4	
Overgeneralization Classification Errors			
Immediate		1,3 > 4	2,3 > 4
Retention		1,3 > 4	
Undergeneralization Classification Errors			
Immediate	1 > 4	1,2,3 > 4	1,2,3 > 4 2 > 1
Retention	2 > 4	1,2,3 > 4	2,3 > 4

*1 > 4 means lesson type (1) favored over lesson type (4).

5. Interpretations

"The use of non-examples which focus the subjects' attention on relevant attributes of the concept was found to be a facilitative instructional technique. Providing a concept definition generally compensated for presenting only the rational set of examples, and negated any possible effects due to

the number of examples given. Presenting the rational set of examples and non-examples was not consistently found to promote concept learning, but presenting the rational set with a concept definition generally was found to be effective. Adding emphasis of relevant attributes to the rational set of examples and non-examples plus a concept definition was generally not found to significantly increase performance."

Abstractor's Notes

1. Several conclusions are based on not finding significant differences.

When significant differences are not found, one normally can only conclude that differences were not found, not that the differences do not exist.

2. The researcher should be commended for relating the work to related research and theory. An excellent conceptual framework is provided.

3. It is unfortunate that neither the full treatments and tests nor even sample items from the treatments and tests were included in the 109 page report. It is most difficult to judge the value of the experiment without such sample items.

4. Multivariate analysis of variance techniques should have been considered. Some explanation for why such techniques were not used should have been provided.

Richard J. Shumway
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EJ 066 774 270 TM 500 713

USE OF A RETENTION INDEX FOR MATHEMATICS INSTRUCTION. Gay, Lorraine R., Journal of Educational Psychology, v63 n5, pp466-72, Oct. 72.

Descriptors--*Retention, *Mathematics Instruction, *Mathematical Concepts, *Teaching Methods, Grade 8, Sex Differences, Computer Assisted Instruction, Tables (Data), [*Retention Index]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Merlyn J. Behr, Northern Illinois University.

1. Purpose

The purpose of this investigation was to evaluate the effectiveness of a preinstruction retention index. This index was designed for utilization in a strategy to maximize recall of mathematical concepts by predicting the idiosyncratic number of examples per mathematical concept required by each student. The study compared three methods of determining amount of practice received on a unit of mathematics in order to determine which method resulted in the most efficient retention of material as measured by a delayed criterion measure. The three methods compared were (a) presenting the student with the number of examples indicated by his score on a preinstruction retention index; (b) allowing students complete choice on each concept as to the number of examples to be received; and (c) presenting each student with a constant number of examples (three) per mathematical concept. It was hypothesized that the retention index group would perform better on measures of both immediate and delayed retention than either the choice group or the fixed group. It was also hypothesized that the retention index group would make fewer acquisition errors.

2. Rationale

Research devoted to the exploration of variables related to retention of learned material has exposed disturbing information: 66% of concepts learned in high school and college courses are forgotten in two years, after a 1-year interval only 33-1/3% of initial algebra material was retained. Such results suggest that considerable time is wasted relearning relevant prerequisite material. Rather conclusive evidence exists to suggest that practice has a positive effect on retention; however, this information has too often been uncritically applied to classroom instruction. For example, the number of examples presented to practice a mathematical concept is typically uniform for all students with inadequate attention being given to individual differences with respect to this variable. The growth of computer-assisted and computer-managed instruction has resulted in a viable strategy for individualization of instruction.

3. Research Design and Procedure

The sample consisted of 53 eighth-grade students of above average

intelligence (27 females and 26 males) from the Florida State University School. Learning materials were presented by an IBM 1500 Computer-Assisted Instruction system. The preinstruction index criterion test was developed by selecting 40 concepts from thirty high school mathematics texts using the criteria of non-familiarity to the subjects and relative independence of the concepts. After pilot work, the number of concepts was reduced to 30. Next the preinstructional retention index was developed by randomly dividing the 30 concepts into five groups. The concepts in each group were presented to subjects followed by 1, 2, 5, 10, or 15 examples, respectively. Sixteen concepts in the instructional unit on polynomials were presented in the same format as the preinstructional retention index -- concept, examples, response frame. Fifteen examples were written for each of the sixteen concepts. The experimental treatment consisted of having all subjects work through the retention index followed by a 1-week delayed retention test. For the 21 subjects randomly selected to the retention index group, retention curves (number of examples by number of correct responses) were plotted from the preinstruction retention index criterion test scores. The number of examples corresponding to the peak of the curve constituted the subjects' retention index scores. All the groups received the same instructional material differing only in the number of examples. All subjects completed the instructional treatment in three sessions. At the end of each session each subject was given an immediate retention measure, and one week following the last session all students were administered the delayed retention test.

4. Findings

The data indicated a sex by treatment interaction; therefore, subsequent analyses included sex as a factor. Data were analyzed using a general linear hypothesis model. The results of a 3 x 2 (treatment by sex) factorial analysis of variance on immediate retention scores confirmed the sex by treatment interaction. Application of the Newman-Keuls procedure showed that for females, the retention index method resulted in significantly better scores than both the choice and fixed methods. Males in the choice group performed significantly better than females in the choice group. Males in the choice group tended to perform better than males in both the retention index and fixed groups. Similar analyses on the delayed retention scores revealed a significant sex by treatment interaction. Females in the retention index group retained significantly more than females in both the choice and fixed groups, and significantly more than males in the retention index and fixed groups, but not more than males in the choice group. Males in the choice group tended to retain more than males in the retention index and fixed groups.

For acquisition scores the results indicate that females in the fixed group made significantly more errors than females in the retention index group; males and females in the retention index group made fewer errors than subjects in the other groups.

5. Interpretations

While previous research on sex differences has indicated that superiority of either sex is a function of the nature of the material, this study suggests

that, similarly, superiority of instructional method is dependent on sex. This study suggests that traditional methods of mathematics instruction allowing all students the same amount of practice are not conducive to promoting retention; it is, therefore, inferred that the high levels of forgetting may be a result of inadequate strategies for providing practice. Note is made of the fact that the average number of examples per concept in the choice group was three -- the same as for the fixed group; however, the number chosen per concept varied. It is reasonable to assume that more examples were chosen for difficult concepts and fewer for easier concepts.

Suggestions for further research include:

a. Conduction of studies with the retention index wherein the concepts are presented with 4, 6, 8, 10, or 12 examples rather than as in this study with 1, 2, 5, 10, and 15.

b. Revision of the index taking item difficulty into account.

c. Revision of the preinstruction retention index and the instructional program to have subjects respond after each example.

Abstractor's Notes

This well-designed and carefully executed study gives some significant information concerning the provision for individual differences in students. This abstractor considers it appropriate to extend this research in a number of ways:

1. Investigate the effects of nonexamples and develop a preinstruction retention index to predict the idiosyncratic number of examples and non examples per mathematical concept.

2. Develop preinstruction acquisition and transfer indices as well as the retention index.

3. Determine whether the preinstruction retention (or other) index score is affected by changing the order (e.g., examples - concept - examples vs. concept - examples) in the presentation.

4. Investigate whether a learner's knowledge of his preinstruction retention (or other) index score has any effect on the number of examples he chooses and then on his subsequent retention. Optimistically one could hope that further research would lead to procedures for developing optimal practice strategies by concept and by student.

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ED 070 657 24 SE 015 466

A STUDY OF THREE CONCEPTS OF PROBABILITY POSSESSED BY CHILDREN IN THE FOURTH, FIFTH, SIXTH AND SEVENTH GRADES. Leffin, W. W., Wisconsin Univ., Madison. Research and Development Center for Cognitive Learning. Spons Agency--National Center for Educational Research and Development (DHEW/OE), Washington, D.C. Pub Date Sep. 71, Note--281p., EDRS Price MF-$0.65 HC-$9.87.

Descriptors--Curriculum, *Doctoral Theses, *Elementary School Mathematics, *Mathematics Education, *Probability, *Research

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Stephen S. Willoughby, New York University.

1. Purpose

To assess the attainment of three concepts of probability by children in grades four through seven who had no formal learning experiences with probability topics, and to relate the level of attainment to I.Q., sex and grade level.

The three concepts were:

I. To identify all possible outcomes of a probability experiment given the points of a finite sample space.

II. To determine the probability of experiments similar to those in item I.

III. To compare situations and determine which had the greater probability.

2. Rationale

Piaget and others have found that children acquire fundamental knowledge of probability concepts during the ages nine through fourteen. Some elementary mathematics programs now incorporate activities involving probability and various recommendations have been made which would accelerate this trend. This study was designed to provide more information regarding the natural acquisition of probability concepts.

3. Research Design and Procedure

The population consisted of all children in the fourth, fifth, sixth and seventh grades of the Wausaw, Wisconsin public shcool district for whom a total I.Q. score on the California Test of Mental Maturity was available - 2,169 children in all. None of the children had been exposed to formal instruction in probability. The district includes rural, suburban and urban communities. The population was partitioned into 2 x 3 x 4 = 24 categories according to sex, I.Q. (71-104, 105-113, 114-144), and grade

(4, 5, 6, 7). From the population for each category, 22 children were selected randomly making a total of 528 children in the sample.

Leffin created one test for each of the three concepts (see "Purpose" section) with 12, 12 and 10 items respectively. All three tests were pencil-and-paper tests with a diagram depicting the situation for each item. The reliability of the tests for the various grade levels ranged from .62 to .81. The tests were administered to the children in groups by Leffin over a three week period during November and December, 1967. Some short instruction was provided at the beginning of each test. Each item was scored either right or wrong.

Multivariate analysis of covariance was used to relate I.Q., sex and grade level, and interactions among these, to performance on the probability tests. The covariates were grade equivalent scores on the three parts of the Stanford Arithmetic Achievement Test (computation, concepts and applications).

4. Findings

Overall mean performances, adjusted for the covariates, were significantly different ($p < .01$) for I.Q., sex and grades.

Univariate analysis of variance showed that high I.Q. groups had higher scores on all three probability tests than the low I.Q. groups, with the middle I.Q. groups scoring between, but overlapping the other groups. Adjusted means for girls were significantly higher on Tests I and II than those for boys while a slight (but not statistically significant) difference favoring the boys existed for Test III. Scores were related to grade level in the obvious way: the higher the grade level, the higher the score; this difference was noted mainly for differences on Test I, but the pattern of differences was the same for all three tests.

Test I was easiest and Test II most difficult for all grades, with items involving probabilities of combinations being most difficult. For example, a problem in which the probability of choosing both cards A and B from a set of four cards marked A, B, C and D when two cards are drawn randomly, stumped 92% of the seventh graders, 98% of the fifth and sixth graders and 99% of the fourth graders. On all other types of items, however, scores were substantially better.

5. Interpretations

Children demonstrated that without formal exposure they had acquired considerable knowledge of the three concepts tested, thus supporting the findings of Piaget and others. It seems reasonable to conclude that with classroom experience they would be able to acquire an even better understanding of the concepts involved and therefore some probability concepts presumably ought to be included in the elementary school curriculum.

Abstractor's Notes

The study was carried out with great care and the report is complete, clear and careful.

The question of what topics from probability ought to be taught at what grade levels, and how much extra help will be needed for children having difficulties is not answered in this study even though there are some indications. For example, it appears that even though children are able to list all combinations for a particular event and are able to determine probabilities when they see a complete sample space, they have trouble combining the two activities. Perhaps this is a problem that can be resolved easily by teaching and perhaps it is a developmental problem that can best be solved by waiting. In any case, a careful study of Leffin's analysis of errors made by the children would be appropriate for any textbook author who includes probability in an elementary school text.

Stephen S. Willoughby
New York University

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ED 070 679 SE 015 514
EFFECTS OF THE ANALYTIC-GLOBAL AND REFLECTIVITY-IMPULSIVITY COGNITIVE STYLES ON THE ACQUISITION OF GEOMETRY CONCEPTS PRESENTED THROUGH EMPHASIS OR NO EMPHASIS AND DISCOVERY OR EXPOSITORY LESSONS. Nelson, Barbara Ann, Wisconsin Univ., Madison. Pub Date 72, Note--155p,
EDRS Price MF-$0.65 HC-$6.58
Descriptors--*Cognitive Processes, Geometric Concepts, Grade 7, *Instruction, Learning, *Learning Characteristics, Learning Theories, Mathematics Education, *Research, *Secondary School Mathematics

Expanded Abstract and Analysis Prepared Espceially for I.M.E. by Zalman Usiskin, University of Chicago.

1. Purpose

To identify two learning styles, learning style being defined as "the interaction between an organismic variable and an instructional treatment."

2. Rationale

Learning style may contribute to a child's success or failure in the classroom. In such a style, The Wisconsin Research and Development Center for Cognitive Learning selects organismic variables by inferring which will interact with operations required by the task, and selects instructional methods which are inferred to be related to the organismic variables.

Operations required by the task were taken from Klausmeier's (1971) model of concept attainment. In this model, discrimination of attributes is important in attaining a formal concept. So the researcher felt "it could be expected that analytic individuals would be able to differentiate the attributes of a concept more easily than would non-analytic individuals."

A second operation involved in Klausmeier's model is that of inferring the concept. Kagan, Pearson, and Welch (1966) found a relationship between scores on tests of inductive reasoning and scores on tests measuring reflectivity and impulsivity. The researcher noted: "If inductive reasoning or inference is important in attaining a formal concept, we would expect that reflective children would learn better than impulsive children when the task required that they discover the relevant attributes by deciding how the nonexamples differ from the examples or how the examples are alike."

3. Research Design and Procedure

Two separate studies (called I and II) were undertaken, one related to each operation in Klausmeier's model. Pilot studies were carried out to evaluate materials, estimate lesson times, test procedures, and assess the appropriateness of the Matching Familiar Figures (MFF) Test to be used in Study II.

Study I: The purpose was to examine the effects of an analytic-global (non-analytic) cognitive style on the immediate acquisition of selected concepts under one of two treatment conditions, verbal emphasis or no emphasis.

From 108 7th graders at one school were chosen the 36 analytic and 36 least analytic, measured by the ETS Hidden Figures Test (1962). These 72 students were stratified by sex and randomly assigned to one of the treatment conditions.

On Day 1, each student individually studied the same introductory lesson at his own rate. On Day 2, one of the treatment lessons was studied, followed by an individually taken Test of Geometry Knowledge when the student was finished. A 2 x 2 x 2 (sex also analyzed) factorial design was used to analyze data from the 67 students present both days.

Study II: The purpose was to examine the effects of a reflective-impulse cognitive style on the immediate acquisition of selected geometry concepts under discovery vs. expository treatment conditions.

From 107 7th graders in classes not involved in Study I (but at the same school), 29 were selected as impulsive, 33 as reflective, based on performance on the MFF Test. Lessons and design were as in Study I, and 53 students were ultimately studied. The same test as in Study I was given at the end of the second lesson.

4. Findings

Study I: Analytic students performed better than non-analytic students. No significant differences were found between emphasis or no-emphasis treatments, and no significant cognitive interactions were found though some data tended to be in the predicted directions.

Study II: No significant differences were found between reflective and impulsive students. Expository lessons resulted in higher performance than discovery lessons. No significant interactions were found and data tended to be opposite the expected directions.

5. Interpretations

The researcher noted, "The failure to obtain a significant interaction between treatment and cognitive style in either study was disappointing." One possible factor may have been that the test or the material was too easy. Other possible difficulties were noted.

The main conclusion is that students who could perceptually separate a simple figure from a complex background (i.e., more analytic students) performed significantly better on a test assessing attainment of geometry concepts, especially on items assessing discrimination of relevant attributes.

"Future research should be focused on identifying the most potent of the organismic variables which affect learning."

Abstractor's Notes

The studies were very carefully designed and clearly reported.

Although the studies were designed "to identify two learning styles," the researcher did not speak to this overall purpose in the discussion and conclusion.

The three geometric "concepts" in this study are identification of rhombuses, parallelograms, and trapezoids. These seem to be three instances of the same idea. Thus the abstract will mislead those who expect a concept to be higher-level than this or who expect that the research covered three different types of content.

The title of the study makes the work seem more generalizable than it is. It should have been noted that the specific content used is as crucial a variable as any in this study. Indeed, given this content, the main conclusion is not unexpected.

Pretests over content are known to be good predictors of learning and might have been used as an additional control.

The first lesson took less than 40 minutes to finish and in Study II, the second lesson averaged either 5 minutes (expository) or 18 minutes (discovery). One questions the wisdom or validity of a 126-item test given after so short a learning period. A shorter test preceding a later retention test might have helped.

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EJ 068 475 270 SE 507 558

THE EFFECTIVENESS OF DISCOVERY AND EXPOSITORY METHODS IN THE TEACHING OF FOURTH-GRADE MATHEMATICS. Olander, Herbert T.; Robertson, Howard C., Journal for Research in Mathematics Education, v4 n1, pp33-44, Jan. 73.

Descriptors--*Elementary School Mathematics, *Instruction, *Research, *Teaching Methods, Discovery Learning, Effective Teaching, Mathematics Education

Expanded Abstract and Analysis Prepared Especially for I.M.E. by William E. Geeslin, University of New Hampshire.

1. Purpose

To evaluate the effectiveness of a discovery approach by comparing it with an expository approach.

2. Rationale

An assumption made in past literature is that "Discovery, or the inductive approach, theoretically fosters a condition in which pupils tend to become active participants in the acquisition of their learnings." In addition, the discovery approach may help a student in "learning to learn." Gage (1963) lists six phases of instruction: (1) initiation of the learning experiences; (2) interjection of teacher knowledge; (3) questioning and answering procedures; (4) appraisal techniques; (5) control of pupil interaction; and (6) use and preparation of materials. Differences between the discovery and expository approaches in each of the above six areas were discussed. For example, one difference between approaches in category (6) is in the discovery approach teachers use concrete objects, models or representative materials to facilitate discovery, while in the expository approach the text is the primary source of ideas.

3. Research Design and Procedure

The study was a 2 x 3 (treatment by test-occasion) design. Treatments were Discovery (D) and Expository (E); test occasions were pre, post, and retention (five weeks after completion of the experiment). Participants in the study were 374 fourth-grade students (187 boys, 187 girls) and 13 teachers (located in 8 buildings). Buildings were assigned randomly to treatments (4 buildings, 7 teachers [7 classes], and 190 students in Treatment D; 4 buildings, 6 teachers [9 classes], and 184 students in Treatment E). The experiment was conducted over 31 weeks (50 minutes of mathematics per day).

Instruments used were: a teacher rating scale to ascertain if teachers were conforming to the assigned treatment (10 ratings of each teacher were made during the study); the Stanford Achievement Test, Form W, Revised 1964 for (a) computation, (b) concepts, and (c) applications (given pre, post, and retention); a Principles and Relationships test (pre, post, and retention); an adaptation of the Dutton and Blum (1968) scale

on attitude toward mathematics (pre and post); and the Otis-Lennon Mental Ability Test, Form J (administered in either the third or fourth grade).

An in-service program before and during the experiment (one hour per week) was conducted to train teachers to use the respective treatments. All teachers used the same text (Silver Burdett Mathematics Series, Grade 4, 1963 Edition, Learning Stages 2 - 7). The text served as the lesson plan for Treatment E; the experimentors developed lesson plans for Treatment D.

One-way analysis of variance was used to analyze teacher ratings. Multiple analysis of covariance (treatment by test occasion by scale) was used to analyze pupil performance on the Stanford Achievement Test scales and Principles and Relationships test. In addition, analyses of pre-post regression lines were used to compare treatments on each achievement scale. Comparison of total mean change was used to examine the attitude scale.

4. Findings

(a) There was a significant difference ($p < .01$) between treatments on the teacher rating scale (in the appropriate direction).

(b) There were significant differences ($p < .05$) between treatments on the Stanford Achievement Test computation scale at posttest and retention test in favor of Treatment E and on the Stanford Achievement Test applications scale at retention test in favor of Treatment D.

(c) There was a significant difference ($p < .05$) in mean change of attitude favoring Treatment D.

(d) Results of the regression analyses indicated interactions between treatments for all three Stanford Achievement Test scales with the regression slope for Treatment D being larger than the regression slope of Treatment E on the computation and application scales and vice versa for the concepts scale.

5. Interpretations

Results of the study indicate that it is possible to get teachers to conform to a specific teaching behavior. There was no clear-cut superiority of either treatment. Attitudes improved more in Treatment D. The regression analyses indicate instruction should be individualized. Conclusions concerning a study are based on the type of data analysis employed.

Abstractor's Notes

Olander and Robertson should be complimented on several aspects of their study: (1) the distinctions between treatments were explained in detail;

(2) teachers were observed to ascertain if they were conforming to the assigned approach; and (3) the study was of sufficient length to determine "educationally" significant results. While one may disagree with these definitions of expository and discovery, he is aware of the distinctions between the two treatments and is more confident that these distinctions actually existed in the study.

It would have been of interest if the researchers had pursued the notion of "learning to learn" as well as final achievement. Olander and Robertson mention that the concrete objects variable may be confounded with the treatment variable. They imply that Treatment E teachers made more of a change from "normal" procedures. These factors, plus the special lesson plans for Treatment D, may have introduced a bias in the experiment in favor of Treatment D.

Olander's and Robertson's conclusions concerning individualized instruction and methods of analysis do not follow from the data (even though they may be true). Several statistical problems also exist in the study. The degrees of freedom used in the analyses should have been number of buildings rather than number of teachers since buildings were assigned to treatments. Degrees of freedom used in the ANCOVA were not reported. No description of the Principles and Relationships test was given, and this test was analyzed in a separate ANCOVA raising a difficulty concerning the exact significance level. No significance levels were attached to the regression analyses leaving doubts concerning the strength of the conclusions from these analyses as well as raising the question of whether the assumptions for the use of ANCOVA were met. The attitude data were analyzed using change scores introducing several well-known problems. While correction of these difficulties is not likely to change the results or conclusions, the difficulties do detract from an otherwise good study.

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ED 067 265 SE 014 715

CONSERVATION OF IDENTITY AND EQUIVALENCE AMONG CHILDREN FROM VARYING SOCIO-ECONOMIC BACKGROUNDS. Pace, Angela, Pub. Date [72], Note--19p.

EDRS Price MF-$0.65 HC-$3.29.

Descriptors--*Conservation (Concept), *Elementary School Mathematics, Grade 1, Grade 2, Kindergarten, Learning, *Mathematics Education, *Number Concepts, *Socioeconomic Influences, Student Characteristics

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Leslie P. Steffe, University of Georgia.

1. Purpose

Pace investigated: (1) whether tasks involving conservation of identity are significantly easier or more difficult than tasks involving conservation of equivalence; (2) if there are important differences in mean chronological ages and intelligence quotients (a) between conservers and nonconservers from low-income backgrounds and (b) between conservers and nonconservers from middle-class backgrounds; (3) if there are significant differences between the two socio-economic groups in the number of conservers on tasks involving conservation of equivalence and on tasks involving conservation of identity.

2. Rationale

Elkind has differentiated between conservation of identity and conservation of equivalence. Pace, applying this distinction to "number conservation," defined conservation of identity and conservation of equivalence. Conservation of identity is where the number of a given set remains unchanged despite rearrangement of its elements; and conservation of equivalence is where once two sets have been placed into one-to-one correspondence, changes in the arrangement of the members of one set or the other in no way affects either the equivalence relation or the cardinal number of each of the equivalent sets.

The view of the above two definitions, Pace declared that no studies exist (1) in which differences among children from low-income and middle-class backgrounds with regard to acquisition of conservation of identity and equivalence were investigated; and (2) in which an attempt was made to determine which of the two concepts (defined above) appear to be easier or more difficult for children from varying socio-economic levels to acquire.

3. Research Design and Procedure

The subjects for the study consisted of 197 kindergarten children, 189 grade one children, and 187 grade two children. Ninety-five of the kindergarten children, 90 of the grade one children, and 89 of the grade two children were from middle-class backgrounds. The children having a low-income background were randomly sampled from three schools serving a predominately middle-class population. The schools were all located in a central New York State city.

The subjects were administered three tests in a 30 day period ending May 5--the Lorge-Thorndike test, Level I Form A or Level II Form A, (depending on the child) and two conservation tests. Test I of the two conservation tests was a test for conservation of equivalence and Test II was for conservation of identity. Test I consisted of three items and Test II of two items. In case of Test I items, the experimenter placed a row (6 to 8) of objects before the child and asked him to get just as many objects from a larger collection (12 to 15) as there were in the row and place them by the original row. The child was then asked if there were just as many objects in the row he constructed as in the original row. If he successfully answered the question, the objects in the row constructed by the child were either pushed together or spread out and the child was asked if there were just as many objects in the row he constructed as in the original row. If the child's answer was affirmative, he was asked, "How can you tell?" If it was negative, he was asked "Are there more _____ than _____?" and "How can you tell?"

In case of Test II items, the examiner placed seven or eight sticks in a row in front of the child and he was asked to count them. The sticks were then rearranged into a pile of two, a pile of three (or four) and two singles. The child was then asked "How many sticks are there now?" and "How do you know?"

Children were classified as conservers or nonconservers on each item of Test I. The conservers were those children who asserted equivalence upon rearrangement of one of the sets and gave a clear explanation. Other responses were categorized as nonconservation responses. The children were also classified as nonconservers on the two items of Test II. If, upon rearrangement of the collection of sticks, the child was able to indicate the correct number of sticks without a recount, he was classified as a conserver. Otherwise, he was classified as a nonconserver.

4. Findings

Within each of the two socio-economic levels, few significant differences were found between mean chronological ages for conservers and nonconservers. A contrast was presented for each test item within each grade, for a total of 15 contrasts within each socio-economic level. For the four significant contrasts that did occur, three occurred for the kindergarten children and the other for the first grade children, all within the low-income category. For all four significant contrasts, the mean age of the conservers was greater than (3 to 4 months) the mean age of the nonconservers.

Analogous contrasts were presented for mean IQ's. For the children in the low-income category, five of the 15 contrasts were significant and for the middle class category, eight of the 15 contrast were significant. For 11 of the 13 significant contrasts, the mean IQ for the conservers was greater than the mean IQ for the nonconservers. These differences in mean IQ's ranged from 4 to 14 IQ points with eight ranging from 4 to 7 and three from 10 to 14. Two of the significant contrast favored the nonconservers, one a difference of 7 IQ points and one difference of 11 IQ points.

Contrast were also presented between the proportion of conservers in the middle-class and low-income categories. A contrast was presented for each conservation test item within each grade for a total of 15 contrasts. In case of equivalence items, two of the nine contrasts were significant ($p < .05$) and three others significant ($p < .01$). Two others approached significance ($p < .10$). All significant contrasts favored the middle-class category. In case of the identity items, only one of six contrasts was significant.

The last set of contrasts presented was between the proportion of conservers on the equivalence items and on the identity items within each socio-economic level. In both cases, the proportion of conservers on the identity items was greater than the proportion of conservers on the equivalence items (.36 vs .20 for the low-income category and .41 vs .32 for the middle-class category).

5. Interpretations

Pace included the following points in her discussion of the study:

(a) Of the two basic ideas underlying the concept of number, "conservation of identity" appears to be easier then "conservation of equivalence."

(b) It can be seen that these tasks (conservation of equivalence) require that the child have an understanding of conservation of identity and of the transitive property of equivalent sets.

(c) This study has shown surprisingly large numbers of children ... have yet to equate the concept of number by the end of the first and second grades.

(d) Teachers ought to make definite efforts to develop the concepts of conservation of identity and conservation of equivalence before requiring children to do much in the way of computation....

Abstractor's Notes

This study, with modification, could be profitably replicated, as the subject of the study is of theoretical importance. In the replication, the following aspects deserve attention. First, there is a difficulty present in the logical relationship of conservation of identity (as defined) and conservation of equivalence (as defined). Obviously, logical identity is a special type of equivalence relation. Just as obviously, physical objects are identical to themselves, but so are numbers such as five. Pace apparently constructed the identity tasks with identity of numbers in mind. The same level of distinction can be made with regard to equivalence relations in general. The set on which an equivalence relation is defined may be a collection of physical objects or numbers, among other objects. The way Pace used equivalence relations had to involve one-to-one correspondence between collections of physical objects. Whether or not number was involved in the definition is not clear. So, on the one hand, we have logical identity

involving numbers (allegedly) and equivalence relations involving one-to-one correspondence between sets of physical objects (presumably). The definitions, then, are not consistent in what is assumed about the set of objects on which the relations are defined. This should be clarified in subsequent studies.

Second, while a statistical test for association may give clues in a search for psychological relations, the two should not be confused. Relationships between conservation of identity and conservation of equivalence need to be searched for at the level of psychological relationships as well as at the level of statistical association.

Third, conservation identity and conservation of equivalence and their interrelations need to be studied across populations with varying characteristics. However, Pace may have been better off to design her study so that various contrasts may have been made within the context of a more inclusive statistical design. To be more specific, if the goal is to attempt to differentiate populations on the basis of a measure of conservation, one should attempt to standardize tasks, thus reducing intra-task variability. The interest here is on characterizing population differences in a gross sense, not task differences. A statistical study is definitely called for, and a well-designed one. If the goal is to look for psychological relationships between tasks--e.g. conservation of identity and nonconservation of equivalence--at the outset, clinical interviews are called for to take into account task subtleties. It is difficult, but not impossible, to take account of both goals in the same study.

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ED 073 944 SE 015 851

EVALUATION OF ALTERNATIVE METHODS OF TEACHING SUBTRACTION OF INTEGERS IN JUNIOR HIGH SCHOOL. FINAL REPORT. Sawyer, Ray C. Eastern Washington State College, Cheney. Spons Agency--National Center for Educational Research and Development (DHEW/OE), Washington, D.C. Regional Research Program. Pub Date Jan 73, Note--170p, EDRS Price MF-$0.65 HC-$6.58.

Descriptors--*Algorithms, Doctoral Theses, *Educational Research, Grade 7, *Integers, Mathematics Curriculum, Mathematics Education, *Mathematics Instruction, Number Concepts, *Secondary School Mathematics, Teaching Methods

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Larry Sowder, Northern Illinois University.

1. Purpose

To compare the effects of 3 methods of teaching subtraction of integers on (a) achievement and retention of the ability to add and subtract integers and (b) computation, concepts, and problem-solving.

2. Rationale

The experience of many teachers suggests that the teaching of subtraction of integers is a trouble spot. A teacher's choice of method is a matter of opinion. There has been no research involving the complement method.

3. Research Design and Procedure

Two sub-studies were performed:

I. Comparison of Related Facts method of subtraction and Complement method of subtraction

$[+7 - -3 = (+7 + +3) - (-3 + +3) = (+7 + +3) - 0 = +7 + +3 = +10]$.

Three teachers, each with 2 classes of randomly assigned seventh graders, volunteered ($n = 140$).

II. Comparison of Systems method of subtraction ($x - y = x + -y$) and Complement method. Two teachers, each with 2 classes of randomly assigned seventh graders, volunteered ($n = 70$). All 5 teachers were given instruction on the material (average time of teacher preparation was 6 hours).

Measures were obtained from two forms of 3 sub-scales of the California Achievement Test (CAT), 1970, Level 4: computation, concepts, and problem-solving. The investigator constructed two forms of scales to measure addition of integers, subtraction of integers, transfer of complement method

to fractional numbers and whole numbers, and subtraction of 3 integers. Tests were administered as follows:

Pre-tests (March)	Treatment	Post-tests	One-month Retention	June Retention
CAT (Form A)	(2 week period)			CAT (B)
Sawyer (Form A)		Sawyer (A)	Sawyer (B)	

The treatment in sub-study I consisted of textbook and/or investigator devised expositions of the integers, addition of integers, either the Related Facts/Complement method of subtraction, and properties of subtraction. Similarly, in sub-study II, units dealing with clock arithmetic, integers, addition of integers, either the Systems/Complement method of subtraction, and properties of subtraction made up the treatment.

4. Findings

ANOVAs showed no statistically significant differences in pre-test scores on computation, concepts, problem solving, addition of integers, and subtraction of integers for the two groups in each sub-study. Achievement ("gain in mean score" from pre- to post- test) was tested by means of "two factor analyses of variance, one factor being repeated measures." Surprisingly (the document is a thesis) raw data on the subjects are not included.

Sub-study I: N.s.d. were found, except that the Related Facts group performed statistically better than the Complement group on achievement in concepts.

Sub-study II: The only significant differences were found in the retention data, Sawyer (B) versus Sawyer (A): The Complement group performed (statistically) better than the Systems group on retention of addition of integers, but less well on retention of subtraction of integers.

Pooled data on Complement groups vs. Related Facts and System groups showed n.s.d. in achievement on subtraction of 3 integers. Complement Ss, although they had instruction on the use of the complement method with whole numbers [e.g., 325-99 = (325 + 1) - (99 + 1) = 326 - 100 = 226], used the method on only 4% of the whole number problems given them.

5. Interpretations

The results suggest that some combination of methods "might be the best solution to the problem of subtraction of integers." Apparently, studying related facts enhances one's growth in concepts, whereas using the complement method enhances one's performance in addition of integers.

Abstractor's Notes

This is the sort of study many teachers would be interested in since its immediate relevance to the classroom is apparent. Consequently, it is extremely important that utmost care be given to the design and the analysis.

The investigator is to be commended in the care exerted in preparing the teachers for the material to be presented and in devising the integers portions of his tests. The desirability of comparing the effects of the 3 treatments simultaneously was perhaps out-weighed by the lack of 3-math-class teachers and the extra preparation required of a teacher using 3 treatments.

There are minor questions as to the rationale for choosing the CAT scales, as to the justifiability of pooling the data in answering some questions, and as to the controls attempted during the post-test to retention and post-test to CAT (B) time intervals. However, a major puzzle to the abstractor is the main analysis. The investigator writes of using a repeated measures analysis, which (in the abstractor's experience) implies that each S underwent each treatment - clearly not the case. The study seems to invite the use of analysis of covariance.

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EJ 067 680 110 SE 507 501

EFFECTIVENESS OF UNIVERSITY OF ILLINOIS COMMITTEE ON SCHOOL MATHEMATICS (UICSM) STRETCHERS AND SHRINKERS AND MOTION GEOMETRY MATERIALS IN IMPROVING ARITHMETIC ABILITY. Trent, John H.; And Others, School Science and Mathematics, v7 n9, pp822-827, Dec 72.

Descriptors--*Curriculum, *Instruction, *Low Achievers, *Research, *Secondary School Mathematics, Fractions, Geometric Concepts, Mathematics Education, Transformations (Mathematics), UICSM

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James M. Moser, University of Wisconsin - Madison.

1. Purpose

To investigate the effectiveness of UICSM courses designed for underachievers, Stretchers and Shrinkers and Motion Geometry, on the arithmetic ability of students taking these courses.

2. Rationale

UICSM has indicated that it feels that the mathematics ability of students taking their courses will improve more than the ability of students studying other mathematics courses. Since very little empirical data have been collected to justify this assertion, the need was felt to carry out experimentation.

3. Research Design and Procedure

Students in a Nevada Junior High School identified as low achievers were randomly assigned to treatment groups. One seventh grade used Stretchers and Shrinkers and another seventh grade used the traditional program of the school. An eighth grade experimental group used Motion Geometry while the control group used the traditional program that had been used for the past three years. A teacher trained in UICSM taught both seventh grades and the eighth grade experimental class while a teacher rated as "outstanding" by the school administration and whose experience approximated that of the other teacher taught the control eighth grade. The Stanford Achievement Test, Advance Battery Form W was administered as both pre and post-test. A t-test was used to test for significance of difference between mean raw score gains between pre and post-tests on subtest scores in Arithmetic Computation, Arithmetic Concepts, and Arithmetic Application as well as total raw score gains.

4. Findings

No significant differences were found in seventh grade scores. In eighth grade a significant difference in favor of the experimental group was found on the Arithmetic Applications subtest score as well as on the total raw score gain. Students in the experimental groups were assessed on their attitudes towards the UICSM materials and were generally found

to be positive toward them.

5. Interpretations

The authors suggest that the reason for the difference in gain scores on Arithmetic Applications is because geometry can be considered as an "application" of arithmetic and because Motion Geometry includes a significant amount or arithmetic.

Abstractor's Notes

The writers do concede that generalizations may be hazardous because the teacher variable may not have been completely controlled. I would be much stronger than that and suggest that teacher variable may, in fact, be the real reason for the noted differences. A comparison of the eighth grade gain scores for the experimental group on all three subsets shows a great deal of similarity in the three scores. In other words, the gain in Arithmetic Applications was not higher than the other two. However, the gain score for the control group on Arithmetic Applications was surprisingly low compared to the other two gain scores on the other subtests. This suggests to me the high probability that the children in the control group suffered from a lack of opportunity to learn whatever was being tested by the subtest on Arithmetic Applications. This is most likely due to the way the teacher taught. This low score on one subtest most likely caused the total raw score gain differences to be significant. Consequently, this study has very little, if any, generalizability.

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EJ 067 549 040 SE 507 497

THE EFFECT OF GEOMETRIC ENRICHMENT EXERCISES ON THE ATTITUDES TOWARD MATHEMATICS OF PROSPECTIVE ELEMENTARY TEACHERS. Wardrop, R.F., School Science and Mathematics v7 n9, pp 794-800, Dec 72.

Descriptors--*Attitudes, *Mathematical Enrichment, *Mathematics Education, *Research, Achievement, College Mathematics, Elementary School Teachers, Geometry, Teacher Education

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Lewis R. Aiken, Guilford College.

1. Purpose

To study the effects of enrichment exercises in geometry on attitudes toward mathematics and achievement in geometry of prospective elementary school teachers.

2. Rationale

Writings and research by other investigators are cited in support of the tenet that instruction affects attitude, and that attitude affects retention and motivation in mathematics. The results of certain studies indicate that enrichment problems have a positive effect on attitude toward mathematics.

3. Research Design and Procedure

Subjects were 111 female students in six sections of a three-hour geometry course required of all elementary education majors at Indiana University. The six sections were divided into three control and three experimental ("enrichment") sections. Three experienced teachers taught one experimental and one control section each, with teachers being assigned at random to sections.

The 111 students were pretested during the first week and post-tested at the end of the semester on the Dutton Attitude Scale and the Geometry Cooperative Mathematics Test. The students in each of the six sections were also implicitly divided into three levels according to their total scores on the Scholastic Aptitude Test (SAT), although this division did not function as a treatment variable.

4. Findings

The first reported analysis consisted of numbers and percentages of students in the experimental and control groups who endorsed each of the 15 items on the Dutton Attitude Scale. On the eight items which are worded in a direction positive toward arithmetic, the experimentals showed increasing

percentages from pre- to post-test on five items and the controls on four items. On the six items worded in a direction negative toward mathematics, the experimentals showed pre- to post-test percentage increases on four and the controls on three items.

The second reported analysis consisted of two treatments (2) x SAT levels (3) x teachers (3) analyses of covariance. In the first ANCOVA, pretest score on the Dutton Attitude Scale was the concomitant variable and posttest score on the scale the dependent variable. None of the main effects nor interactions was statistically significant. In the second ANCOVA, pretest score on the geometry achievement test was the concomitant variable and posttest score the dependent variable. Only the main effect of the "teacher" variable was statistically significant.

5. Interpretations

The authors'conclusions are that: neither attitude toward mathematics nor achievement in geometry was significantly affected by the geometric enrichment exercises or by the interaction of the two instructional treatments with scholastic aptitude or differences among the three teachers. Due to the lower performances of the experimental and control sections taught by one instructor however, the instructor variable had a statistically significant effect on student achievement in geometry.

Abstractor's Notes

This investigation contains several methodological flaws, which, although by no means unique, make interpretation of the results difficult. To begin, the work "arithmetic" in the Dutton Scale should have been changed to "mathematics" before it was ever administered to the students. The scale values of the 15 items on the Dutton Scale are not directly transferrable to this investigation in any case, so the use of the word "mathematics" would have done no great violence to the results. Secondly, teachers rather than students were assigned at random to sections, so the reported ANCOVAs are incorrect. Coupled with the fact that the numbers of Ss in each of the 18 cells were probably unequal, an unweighted means analysis of regressed gain scores is called for. Even if the author had decided to go ahead with the ANCOVA, he should have reported the results of the preliminary tests for common slope. Also, since SAT was not an experimental variable, but rather a correlated or organismic variable, there is no justification for an F test of SAT main effects. It probably makes some sense, however, to look at the effects of the interaction between SAT and the treatment conditions. Finally, the author refers to a multi-T-test as having been run on the teacher variable in the second ANCOVA. Does he mean multiple t tests? If so, he's wrong, because the comparisons are not independent. A Scheffé test or similar post hoc analysis should have been conducted on the teacher means.

To nit-pick a bit, on page 798 it is concluded that "the null hypothesis was accepted." Failure to reject the null hypothesis does not imply its acceptance; one doesn't prove the null hypothesis in Fiserian inference. Also, there is a division error on line 7 of Table 3; .67 should read .067 or .07.

One thing that I missed terribly in this investigation is a table of group means. I did a bit of arithmetic on the frequencies and scale values in Table 2 and found the following means on the Dutton Scale:

	Experimental	Control
Pretest	5.52	6.15
Posttest	6.25	6.15

Thus, although there was a differential improvement in the attitudes of the experimental groups, it was not statistically significant. At the very least this suggests that a more carefully conducted investigation might reveal that enrichment exercises do make a difference in attitude toward mathematics. But it is recommended that when this follow-up study is conducted it should not employ the Dutton Scale. When the 15 items on the Dutton Scale are placed in rank order according to Scale value, adjacent items are a long way from equidistant. This is a violation of Thurstone's equal intervals requirement. In addition, I suspect that the ambiguity indices of many of these items are rather high.

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EJ 067 210
THE ABILITY OF FIRST-, SECOND-, AND THIRD-GRADE PUPILS TO IDENTIFY OPEN ADDITION AND SUBTRACTION SENTENCES FOR WHICH NO SOLUTION EXISTS WITHIN THE SET OF WHOLE NUMBERS. Weaver, J.F., School Science and Mathematics, v72 n8, pp679-691, Nov 72.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Donald J. Dessart, The University of Tennessee, Knoxville.

1. Purpose

To determine how well pupils of the first, second, and third grades recognize certain open sentences with empty solution sets, when open sentences of this type have not been studied systematically by these students.

2. Rationale

Most programs in mathematics designed for young children include open sentences of the following kinds, each of which possesses a non-empty solution set in the whole numbers (W):

$4 + \square = 10$ $\qquad$ $11 - 3 = \square$

$9 = \square + 2$ $\qquad$ $5 = 8 - \square$

Such programs rarely include open sentences of the following types whose solution sets in W are empty:

$\square + 9 = 6$ $\qquad$ $\square = 5 - 8$

$4 = 10 + \square$ $\qquad$ $7 - \square = 11$

An interesting question arises in regard to how well students who have been exposed to finding solution sets for open sentences of the first type shown above will treat open sentences of the second type especially when these pupils have not been given explicit instruction concerning the second type of open sentence.

3. Research Design and Procedure

Using the equation, $a \circ b = c$, and its symmetric equivalent, $c = a \circ b$, with two operations (+ and -) and three place-holder positions (a, b, or c) a total of 12 (2 x 2 x 3) open sentence forms were constructed as shown:

1) $a + b = \square$

2) $\square = a + b$

3) $a + \square = c$

4) $c = a + \square$

5) $\square + b = c$

6) $c = \square + b$

7) $a - b = \square$

8) $\square = a - b$

9) $a - \square = c$

10) $c = a - \square$

11) $\square - b = c$

12) $c = \square - b$

In each of these 12 forms, a, b, or c may be replaced by specific whole numbers so that non-empty solution sets exist in W; and in each of 8 forms (excluding 1, 2, 11, and 12) a, b, or c may be replaced by specific whole numbers so that the solution sets in W are empty. Consequently a total of 20 forms may be constructed.

Specific examples of these 20 forms were assigned to four tests (Tests 11, 12, 13, 14) in such a way as to insure a balance in the four tests among the following factors: (1) $a \circ b = c$ or $c = a \circ b$, (2) $\circ$ specified as + or -, (3) the position of $\square$ in the sentence, and (4) the existence of an empty or a non-empty solution set. These examples included the set of basic addition and subtraction facts (excluding doubles) having sums between 10 and 18 or differences less than 10. These items were arranged randomly within each test with the exception that the first item of each test is an open sentence for which the solution set is non-empty.

At this point an innovative variation was introduced into the tests. Instead of using the usual open box ($\square$), a solid frame (■) was used in each open sentence with the pupils provided the instruction that a whole number may be "hiding under the box." If the pupil decided that a whole number satisfying the equation was hiding under the box, he was instructed to write its numeral in the blank to the right of the open sentence; but if no such number was hiding under the box (because the solution set was empty), he was to mark the appropriate blank with a large "X".

The tests were administered to at most two classes from each of the first, second, and third grade levels of schools participating in a National Science Foundation Cooperative College - School Science project in mathematics. This represented about 75 percent of the elementary schools in a particular school system. The four tests were distributed randomly in each of the classes, and the teachers after completing a training session administered the tests. No teacher administered a test to her own class. The testing was done in March and April of 1970 in 29 schools at times that it was determined the pupils of each school had completed comparable materials. Thus, it could be assumed that each pupil had been exposed to the same pre-testing instructional materials.

4. Findings

No formally stated hypotheses were tested nor was any attempt made to generalize the results of this investigation to a larger population. Only normative data were reported.

Data were discussed in two sections, A and B. A contained the overall mean percent of correct responses (X used correctly) for each of the forms 3 through 10 and for each of the grade levels 1, 2, 3. In general, the mean percent of correct responses for forms 3-6 increased from grade level to grade level with the exception of form 5 ($\square + b = c$) which increased from grade 1 (41.9) to grade 2 (59.6) but decreased slightly in grade 3 (58.9). Furthermore, form 6 ($c = \square + b$) appeared to be most difficult at grade levels 1 and 2 (had the smallest mean percent of correct responses) but was the least difficult at grade 3 (had the greatest mean percent of correct responses). The mean percent of correct responses for forms 7-10, combined, was about the same for each grade level, i.e. grade 1 (41.6), grade 2 (41.7), and grade 3 (41.9).

Part B contained mean percent of incorrect responses (X used incorrectly) for pupils of grades 1, 2, 3 and for each of the forms 1 through 12. For forms 1-6 (addition sentences), the mean percent of incorrect responses decreased from grade level to grade level, i.e., progressive improvement was observed from grade to grade with the exception of form 2 ($\square = a + b$) which decreased from grade 1 (8.2) to grade 2 (1.5) but increased very slightly to 1.8 in grade 3. In form 4 ($c = a + \square$), 22.5 percent of the first graders used the X incorrectly on Test 11 but only 8.9 percent used it incorrectly on Test 14. This was the most marked difference observed between similar forms on different tests. For forms 7-12 (subtraction sentences), the mean percent decreased from grade level to grade level with the exception of form 11 ($\square - b = c$) of Test 12 and form 12 ($c = \square - b$) of Tests 11 and 14, in which the percent of incorrect uses was about the same (approximately 25 percent) for each grade level.

5. Interpretations

The author concluded that the data suggested that the students were not completely lost in dealing with open sentences having empty solution sets even though this concept had not been explicitly taught to them. He also concluded that the students' ability to identify "no solution" situations was a "rather variable phenomenon" associated with factors of grade level and sentence form.

Furthermore, it appeared that the idea of "no solution" was handled more satisfactorily in the addition than the subtraction sentences. The author conjectured that part of this difficulty may have been due to the fact that commutativity is stressed for addition but non-commutativity of subtraction is often omitted. Consequently, students may erroneously conclude, for example, that $4 - 7 = \square$ has the same solution set as $7 - 4 = \square$.

In structured interviews with a small sample of students, it was also found that "a nontrivial number of pupils interviewed read a nontrivial number of open sentences backwards." Such a reading will not lead to errors in determining the solution sets for addition sentences but can result in the determination of erroneous solution sets for subtraction sentences.

Abstractor's Notes

This study is an excellent example of an investigation that has appeal to both researchers and classroom practitioners. It considers a basic question that has relevancy for the task of organizing mathematical instruction in the elementary schools. The author had wisely chosen to report normative statistical measures rather than to employ unwarrented inferential techniques. Means were appropriately provided, but the reader might have gained a more complete picture if measures of variability, such as standard deviations, had also been presented.

It was noted that the pupils of the study had not been exposed to formal instruction concerning number sentences that possessed empty solution sets, but it was not reported whether or not the symmetric property of number sentences and its implications had been systematically studied by these students. It would seem that this question would have a close relationship to the "reading backwards" problem cited in the report.

The innovation of using a solid frame instead of an open frame with the understanding that a whole number may be "hiding under the box" would undoubtedly appeal to the inquisitive nature of elementary school pupils and consequently seems to have considerable merit. Some educators, however, would criticize this technique because it may lead students into the "old fashioned" notion that the only function of a variable is to hold the place in a number sentence for the number or numbers which make the sentence true rather than as a placeholder for any member of the replacement set.

This study is certainly an interesting one and will probably stimulate other researchers on related questions.

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EJ 067 203 450 SE 506 933
DECIMAL-COMMON FRACTION SEQUENCE VERSUS CONVENTIONAL SEQUENCE. Willson, George H., School Science and Mathematics, v72 n7, pp589-592, Oct. 1972.
Descriptors--*Decimal Fractions, *Elementary School Mathematics, *Mathematics Education, *Research, Curriculum, Fractions, Grade 5, Teaching Methods

Expanded Abstract and Analysis Prepared Especially for I.M.E. by H. Laverne Thomas, State University of New York College at Oneonta.

1. Purpose

The purpose of the study was "to compare arithmetic achievement in samples of children taught arithmetic by the decimal-common fraction sequence versus the conventional sequence."

2. Rationale

The investigator was concerned with basic questions of placement and emphasis for decimal and common fractions in the elementary school program. This concern was with particular regard to these questions in the context of contemporary mathematics programs. Although no specific reference is made, the study reported is in the general realm of research on appropriate instructional sequencing.

3. Research Design and Procedure

The design is apparently that of a two treatment pretest-posttest model. Subjects were 112 fifth grade students (56 male, 56 female) in the same elementary school who were randomly assigned to one of four classes prior to the study. Four teachers were randomly assigned to class and to treatment so that two treatment groups, each consisting of two classes with different teachers resulted. The treatment variable was that of instructional sequence: conventional (common - decimal fraction) and experimental (decimal - common fraction).

Teachers were especially selected for the study and were experienced with the common-decimal fraction sequence but not the decimal-common fraction sequence. These teachers received a special teachers' manual and guidance and consultation from the investigator. Adaptations of a standard contemporary text constituted the instructional materials.

Criterion data on achievement was provided through a standardized grade 5 arithmetic achievement test.

Total achievement data and achievement data in several subareas was obtained from this test. Two analyses were made. First, differences between pretest and posttest scores on total achievement within treatment group was analyzed by two-tailed t-tests. Second, analysis of covariance was employed to test differences between treatment groups on total achievement

and on the various subareas (problem solving, computation, concepts, etc.).

4. Findings

Highly significant differences ($\alpha = .001$) were found between pretest and posttest scores for each treatment group. No significant differences were found between groups for total achievement or for any subarea.

5. Interpretations

Fifth grade elementary school students can make a significant gain in achievement on decimal and common fractions when taught by either the decimal-common fraction sequence or the common-decimal fraction sequence. Also, the two sequences result in equal performance on total achievement and on each of the associated subareas of achievement. The investigator further observes that a greater raw score gain was made by students taught by the decimal-common fraction sequence. From this the investigator speculates that significant differences might be found if the scope of the investigation were to be widened to include multiplication and division. Further, would the decimal-common fraction sequence exhibit greater increase in achievement if "equal time" were to be given to decimal fractions in the prior informal treatment of fractions?

Abstractor's Notes

The brevity with which the procedures and analysis of data are presented in this report raises a number of questions. How were the modifications made of the textbook materials to minimize reliance on previous background with common fractions in the decimal-common fraction sequence? What was the duration of the instructional period? When were the pretest and the posttest administered relative to the instructional period? What form of the t-test was employed to analyze gains? What were the covariates in the analysis of covariance?

These questions, and questions as to what extent the two treatments represent truly different instructional sequences, weaken any conclusions that might be drawn from the study. Further, with a "no significant differences" result, there is nothing to suggest advantages in the altered sequence comparable to the economy of concepts employed in the traditional sequence.

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EJ 067 216 450 SE 507 297

A DESIGN FOR CLASS TESTING MATHEMATICS TEXTBOOK MATERIALS. Zahn, Karl G., Two-Year College Mathematics Journal, v3 n2, pp.29-32, Fall 72.

*College Mathematics, *Instruction, *Mathematics Education, *Research, *Textbook Research, Class Organization, Curriculum, Individualized Instruction, Instructional Materials, Student Motivation

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Gerald Kulm, Purdue University.

1. Purpose

The study was conducted to test the "teachability" of a college liberal arts mathematics textbook. The null hypothesis was that students having average or below average backgrounds in high school math will not gain significantly in mathematical knowledge through extrinsically motivated study of The Four Roles of Mathematics by Henderson and Johnson.

2. Rationale

The primary assumption was that the teacher effect should be reduced in order that students learn only from the materials being tested. No previous research literature was cited.

3. Research Design and Procedure

The subjects were 19 students in an 8 week summer "liberal arts" math course, meeting 3 hours a week. The Ss were given a multiple-choice pretest, then put into small heterogeneous groups (3-5) on the basis of pretest scores. Each class period followed the same procedure:

a. Corrected quizzes from previous class were returned.
b. Group discussion of returned quizzes and assignment for the day.
c. A 15 minute quiz. Each group member received the average grade of his group.
d. Assignments for the following day handed out.

At the end of the course, a posttest, identical to the pretest, was given to the subjects. Subjects were told that the posttest, the group quizzes, and a separate final examination were to be used in determining their final grade.

The significance of the difference between the pretest and posttest means was determined by a t-test.

4. Findings

The mean of the pretest scores was 2.8 and the mean of the posttest scores was 13.9. The difference between the means was significant ($p < .001$), thus the null hypothesis was rejected.

5. Interpretations

The conclusion was that the students did make a significant gain using the materials through extrinsically motivated study. The students not only enjoyed the class sessions but met outside of class to discuss assignments.

The method is suggested not only for testing new materials, but for individualizing mathematics instruction.

Abstractor's Notes

Perhaps the most useful aspect of this study is the suggestion that the method be used to individualize mathematics instruction. There were some serious problems with the research design and procedure.

a. The study did not achieve its purpose of testing the teachability of the materials since the textbook was not taught but read by the students. Even the readability can be questioned since, in the heterogeneous groups, one good reader could have interpreted his comprehension of the material to the other group members.

b. The small number of subjects, the absence of a control group, and the analysis of gain scores detract further from the value of the study.

A number of empirical results have been obtained regarding the readability of mathematical textbook material. Hopefully, researchers and teachers will soon begin to use these readability results in assessing textual meterials.

Gerald Kulm
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